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DETERMINISTIC INVENTORY CONTROL MODEL FOR DETERIORATING ITEMS UNDER PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT:

In this paper an inventory control model for deteriorating items is considered. The mathematical model is derived in the presence of trade credit period with permissible delay in payments, but expiry of the time will charge some interest. Here price discount is allowed for deteriorated items. In this model demand rate is linear and holding cost is known and constant. The model is solved analytically by maximizing the total profit. A numerical example is given to illustrate this model.

.**Key words**: inventory, trade credit period, demand, holding cost.

1. INTRODUCTIN

In the real world, the seasonal goods like fashion goods have been attended where they have been ordered at a period and have sold at a given duration. The real life situation becomes more complicated when the inventories are subject to deterioration. The several researchers developed the inventory model for deteriorating items with time dependent demand rate.

Generally it is assumed that the buyer must pay for the items as soon as he receives them from the supplier, but in reality supplier will allow a certain fixed period called credit period. The credit period reduces the buyers' cost of holding stock because it reduces the amount of capital invests in stock for the duration of the permit period. Goyal S.K 1985[5] explored a single item economic order quantity model under conditions of permissible delay in payments. Chung K.J and Hwang Y.F 2002[3] studied the same model. Goyal S.K 1985 [5] developed an alternative approach to finding a theorem to determine the EOQ under conditions of permissible delay in payments and Aggarwal S.P and Aggi C.K 1992 [1] extended and analyzed the credit financing in economic ordering policies of deteriorating items in the presence of trade credit using a DCF approach. Chang H.J Dye C.Y 2001 [2] present an EOQ model with deteriorating items under inflation when the supplier provides a permissible delay of payments for a large predictor, that is greater than or equal to the predetermined quantity. Jaggi.C.K. and Aggarwal.K.K, Goel.S.K (2006) [6] developed an Optimal order policy for deteriorating items with stock-dependent demand, inflation induced demand. Neetu, Arun Kumar Tomer,Sapna Mahajan 2010 [7] developed a model for the deteriorating items in the presence of trade credit period under discounted cash flow approach with permissible delay in payments.

In this paper, we developed a model in which demand of a product is assumed as a decreasing function of time and shortages are allowed. In this model deteriorating items is considered in the presence of trade credit period with permissible delay in payments. The numerical examples have been given to illustrate the model.

2. ASSUMPTIONS AND NOTATIONS

2.1 ASSUMPTIONS:

- Replenishment time is instantaneous.
- The demand rate is linear.
- Price discount for deteriorating items.
- The holding cost is known and constant.
- Lead time is zero.
- Shortages are allowed.

2.2 NOTATIONS

D(t) -- The Demand rate per unit time (a+bt), a > 0, 0 < b < 1.
θ -- The constant deterioration rate per unit time.
h-- The holding cost per unit.
s--The selling price per unit item.
Q–Theordering quantity.
A-- The ordering cost per unit.
T--The length of the cycle time.
c_1--The inventory shortage cost per unit time.
t_0-- The time at which the inventory level reaches zero.
P --The purchasing cost of an item.
S -- The shortages of quantity per cycle.
I_e -- The interest earned per unit time.
I_p -- The interest payable per unit time. $I_p > I_e$
M-- The permissible delay in selling the product.
TP_1-- The totalprofit per unit time (case-I)
TP_2--The total profit per unit time (case-II)

3. MATHEMATICAL FORMULATION:

In this mathematical model, demand rate is linear andrate of deterioration is constant. The deterioration is possible when the products are available in the stock. Depletion of the inventory occurs due to demand as well as due to deterioration. At time t_0 inventory level goes to zero and shortages occur.

$$\frac{dI(t)}{dt} + \theta I(t) = -(a+bt) \qquad 0 \le t \le t_0 \qquad ------(1)$$

$$\frac{dI(t)}{dt} = -(a+bt) t_0 \le t \le T \qquad ------(2)$$

Boundary conditions are I(0) = Q, $I(t_0)$ = 0, I(T) = S. Solving the differential equations, we get the inventory level as follows.

$$I(t) = -at + Q(1-\theta t) \;\; 0 \le t \le t_0 ------(3)$$

$$Q = \frac{at_0}{1-\theta t_0} \qquad ------(4)$$

$$I(t) = \left[a(T-t) + \frac{b}{2}(T^2 - t^2) - S\right] t_0 \le t \le T ------(5)$$

$$S = \left[a(T-t_0) + \frac{b}{2}(T^2 - {t_0}^2)\right] ----(6)$$

Based on the assumption and descriptions of the model, the total profit (TP), include the following elements.

The ordering cost OC=A $----(7)$

The holding cost $HC = h\int_0^{t_0} I(t)dt$

$$HC = h\left[-\frac{a{t_0}^2}{2} + Q\left(t_0 - \frac{\theta {t_0}^2}{2}\right)\right] ----(8)$$

The shortage cost $SC = c_1 \int_{t_0}^{T} I(t)dt$

$$SC = c_1\left[\frac{a}{2}(T-t_0)^2 + \frac{b}{2}\left(\frac{2T^3}{3} - T^2 t_0 - \frac{{t_0}^3}{3}\right) - S(T-t_0)\right] ----(9)$$

The purchasing cost$PC = P * (Q+S)$ $----(10)$

$$PC = P * \left\{\frac{at_0}{1-\theta t_0} + \left[a(T-t_0) + \frac{b}{2}(T^2 - {t_0}^2)\right]\right\}$$

The price discount is

$$PD = (s - 0.1s)\theta \frac{at_0}{1-\theta t_0} \qquad ----(11)$$

The sales revenue is

$$SR = s\int_0^T \mathrm{D(t)dt}$$
$$\mathrm{SR} = \mathrm{s}\left(\mathrm{aT} + \frac{\mathrm{bT}^2}{2}\right) ----(12)$$

Here two cases may arise depending upon the value of M.

3.1 Case-I $M \leq t_0$

In this case, the length of period with positive stock is larger than the permissible delay period, the buyer can use the sales revenue to earn interest at an annual rate I_e in $(0, t_0)$,
The interest earns IE_1 is

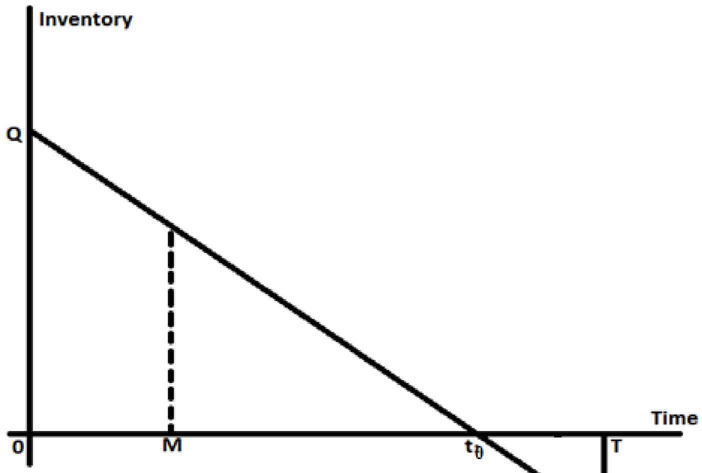

Fig 1

$$\mathrm{IE_1} = \mathrm{sI_e}\int_0^{\mathrm{M}} (\mathrm{a} + \mathrm{bt})\mathrm{dt}$$
$$\mathrm{IE_1} = \mathrm{sI_e}\left[\mathrm{aM} + \frac{\mathrm{bM}^2}{2}\right] \qquad ----(13)$$

The interest payable is

$$\mathrm{IP_1} = \mathrm{PI_p}\int_{\mathrm{M}}^{\mathrm{t_0}} (\mathrm{a} + \mathrm{bt})\mathrm{dt}$$
$$\mathrm{IP_1} = \mathrm{PI_p}\left[\mathrm{Q(t_0 - M)} - \left(\frac{\mathrm{a} + \theta}{2}\right)(\mathrm{t_0}^2 - \mathrm{M}^2)\right] ----(14)$$

3.2 Case-II $M > t_0$

Since $M > t_0$the buyer earns interest at an annual rate I_e, during the period (0, M). Interest earn in this caseis denoted by IE_2 and is given by

Fig 2

$$\mathrm{IE_2} = \mathrm{sI_e}\int_0^{t_0} (\mathrm{a}+\mathrm{bt})\mathrm{dt}$$

$$\mathrm{IE_2} = \mathrm{sI_e}\left[\left(\mathrm{at_0} + \frac{\mathrm{bt_0}^2}{2}\right) + (\mathrm{M}-\mathrm{t_0})(\mathrm{a}+\mathrm{bt_0})\right] \qquad ----(15)$$

The interest payable is $\mathrm{IP_1} = 0$ $\qquad ----(16)$

$$\mathrm{TP_1} = \frac{1}{\mathrm{T}}[\mathrm{SR}-\mathrm{OC}-\mathrm{PC}-\mathrm{HC}-\mathrm{SC}-\mathrm{PD}-\mathrm{IP_1}+\mathrm{IE_1}] ----(17)$$

$$\mathrm{TP_2} = \frac{1}{\mathrm{T}}[\mathrm{SR}-\mathrm{OC}-\mathrm{PC}-\mathrm{HC}-\mathrm{SC}-\mathrm{PD}-\mathrm{IP_2}+\mathrm{IE_2}] ----(18)$$

Substituting thevalues from equations (3)to (15) in equations (17) and (18), we get the total profit per unit.

$$\begin{aligned}\mathrm{TP_1} = \frac{1}{\mathrm{T}}\Bigg[& \mathrm{s}\left(\mathrm{aT}+\frac{\mathrm{bT}^2}{2}\right) - \mathrm{A} - \mathrm{P}*\left\{\frac{\mathrm{at_0}}{1-\theta\mathrm{t_0}} + \left[\mathrm{a}(\mathrm{T}-\mathrm{t_0}) + \frac{\mathrm{b}}{2}(\mathrm{T}^2-\mathrm{t_0}^2)\right]\right\} \\ & - \mathrm{h}\left[-\frac{\mathrm{at_0}^2}{2} + \frac{\mathrm{at_0}}{1-\theta\mathrm{t_0}}\left(\mathrm{t_0} - \frac{\theta\mathrm{t_0}^2}{2}\right)\right] \\ & - \mathrm{c_1}\left[\frac{\mathrm{a}}{2}(\mathrm{T}-\mathrm{t_0})^2 + \frac{\mathrm{b}}{2}\left(\frac{2\mathrm{T}^3}{3} - \mathrm{T}^2\mathrm{t_0} + \frac{\mathrm{t_0}^3}{3}\right) \right. \\ & \left. - (\mathrm{T}-\mathrm{t_0})\left[\mathrm{a}(\mathrm{T}-\mathrm{t_0}) + \frac{\mathrm{b}}{2}(\mathrm{T}^2-\mathrm{t_0}^2)\right]\right] - (s-0.1s)\theta\frac{\mathrm{at_0}}{1-\theta\mathrm{t_0}} \\ & - \mathrm{PI_p}\left[\mathrm{Q}(\mathrm{t_0}-\mathrm{M}) - \left(\frac{\mathrm{a}+\theta}{2}\right)(\mathrm{t_0}^2-\mathrm{M}^2)\right] + \mathrm{sI_e}\left[\mathrm{aM}+\frac{\mathrm{bM}^2}{2}\right]\Bigg] ------ (19)\end{aligned}$$

$$\begin{aligned}\mathrm{TP_2} = \frac{1}{\mathrm{T}}\Bigg[& \mathrm{s}\left(\mathrm{aT}+\frac{\mathrm{bT}^2}{2}\right) - \mathrm{A} - \mathrm{P}*\left\{\frac{\mathrm{at_0}}{1-\theta\mathrm{t_0}} + \left[\mathrm{a}(\mathrm{T}-\mathrm{t_0}) + \frac{\mathrm{b}}{2}(\mathrm{T}^2-\mathrm{t_0}^2)\right]\right\} \\ & - \mathrm{h}\left[-\frac{\mathrm{at_0}^2}{2} + \frac{\mathrm{at_0}}{1-\theta\mathrm{t_0}}\left(\mathrm{t_0} - \frac{\theta\mathrm{t_0}^2}{2}\right)\right] \\ & - \mathrm{c_1}\left[\frac{\mathrm{a}}{2}(\mathrm{T}-\mathrm{t_0})^2 + \frac{\mathrm{b}}{2}\left(\frac{2\mathrm{T}^3}{3} - \mathrm{T}^2\mathrm{t_0} + \frac{\mathrm{t_0}^3}{3}\right) \right. \\ & \left. - (\mathrm{T}-\mathrm{t_0})\left[\mathrm{a}(\mathrm{T}-\mathrm{t_0}) + \frac{\mathrm{b}}{2}(\mathrm{T}^2-\mathrm{t_0}^2)\right]\right] - (s-0.1s)\theta\frac{\mathrm{at_0}}{1-\theta\mathrm{t_0}} \\ & + \mathrm{sI_e}\left[\left(\mathrm{at_0}+\frac{\mathrm{bt_0}^2}{2}\right) + (\mathrm{M}-\mathrm{t_0})(\mathrm{a}+\mathrm{bt_0})\right]\Bigg] ----- (20)\end{aligned}$$

The optimal value of T and t_0 which maximizes TP_1 and TP_2 can be obtained bydifferentiating(19),(20) with respect to T and $t_0$and by using maxima and minima method.

$$\frac{\partial TP_1}{\partial T} = -\frac{1}{T^2}\left[-A - P*\left[at_0\left(\frac{\theta t_0}{1-\theta t_0}\right) - \frac{bt_0{}^2}{2}\right] - h\left[-\frac{at_0{}^2}{2} + \frac{at_0}{1-\theta t_0}\left(t_0 - \frac{\theta t_0{}^2}{2}\right)\right]\right.$$
$$- c_1\left[\frac{3at_0{}^2}{2} + \frac{2at_0{}^3}{3}\right] - (s-0.1s)\theta\frac{at_0}{1-\theta t_0}$$
$$- PI_p\left[\frac{at_0}{1-\theta t_0}(t_0 - M) - \left(\frac{a+\theta}{2}\right)(t_0{}^2 - M^2)\right] + sI_e\left[aM + \frac{bM^2}{2}\right]\Bigg] + \frac{b(P+s)}{2}$$
$$+ c_1\left[\frac{a}{2} + \frac{Tb}{3}\right] = 0 \qquad ------(21)$$

$$\frac{\partial TP_1}{\partial t_0} = \frac{1}{T}\left[-Pt_0[2a\theta - b] - h\left(at_0 + \frac{3a\theta t_0{}^2}{2} - 2a\theta t_0{}^3\right) - c_1[aT + bt_0(T - t_0)]\right.$$
$$\left. - (s-0.1s)\theta a(1+2\theta t_0) - PI_p\left[at_0 - aM + 3a\theta t_0{}^2 - 2a\theta M t_0 - \theta t_0\right]\right]$$
$$= 0 \qquad ------(22)$$

$$\frac{\partial TP_2}{\partial T} = -\frac{1}{T^2}\left[-A - P*\left[at_0\left(\frac{\theta t_0}{1-\theta t_0}\right) - \frac{bt_0{}^2}{2}\right] - h\left[-\frac{at_0{}^2}{2} + \frac{at_0}{1-\theta t_0}\left(t_0 - \frac{\theta t_0{}^2}{2}\right)\right]\right.$$
$$- c_1\left[\frac{3at_0{}^2}{2} + \frac{2at_0{}^3}{3}\right] - (s-0.1s)\theta\frac{at_0}{1-\theta t_0}$$
$$+ sI_e\left[\left(at_0 + \frac{bt_0{}^2}{2}\right) + (M - t_0)(a + bt_0)\right]\Bigg] + \frac{b(P+s)}{2} + c_1\left[\frac{a}{2} + \frac{Tb}{3}\right]$$
$$= 0 \qquad ------(23)$$

$$\frac{\partial TP_2}{\partial t_0} = \frac{1}{T}\left[-Pt_0[2a\theta - b] - h\left(at_0 + \frac{3a\theta t_0{}^2}{2} - 2a\theta t_0{}^3\right) - c_1[aT + bt_0(T - t_0)]\right.$$
$$\left. - (s-0.1s)\theta a(1+2\theta t_0) + sI_e(Mb - bt_0)\right] = 0 \qquad ------(24)$$

$$\frac{\partial^2 TP_1}{\partial T^2} = \frac{2}{T^3}\left[-A - P*\left[at_0\left(\frac{\theta t_0}{1-\theta t_0}\right) - \frac{bt_0{}^2}{2}\right] - h\left[-\frac{at_0{}^2}{2} + \frac{at_0}{1-\theta t_0}\left(t_0 - \frac{\theta t_0{}^2}{2}\right)\right]\right.$$
$$- c_1\left[\frac{3at_0{}^2}{2} + \frac{2at_0{}^3}{3}\right] - (s-0.1s)\theta\frac{at_0}{1-\theta t_0}$$
$$- PI_p\left[\frac{at_0}{1-\theta t_0}(t_0 - M) - \left(\frac{a+\theta}{2}\right)(t_0{}^2 - M^2)\right] + sI_e\left[aM + \frac{bM^2}{2}\right]\Bigg] + \frac{bc_1}{3}$$
$$< 0 \qquad ------(25)$$

$$\frac{\partial^2 TP_1}{\partial t_0{}^2} = \frac{1}{T}\left[-P[2a\theta - b] - h(a + 3a\theta t_0 - 6a\theta t_0{}^2) - c_1[bT - 2bt_0] - (s-0.1s)2\theta^2 a\right.$$
$$\left. - PI_p[a + 6a\theta t_0 - 2a\theta M - \theta]\right] < 0 \qquad ------(26)$$

$$\frac{\partial^2 TP_2}{\partial T^2} = \frac{2}{T^3}\left[-A - P*\left[at_0\left(\frac{\theta t_0}{1-\theta t_0}\right) - \frac{bt_0{}^2}{2}\right] - h\left[-\frac{at_0{}^2}{2} + \frac{at_0}{1-\theta t_0}\left(t_0 - \frac{\theta t_0{}^2}{2}\right)\right]\right.$$
$$- c_1\left[\frac{3at_0{}^2}{2} + \frac{2at_0{}^3}{3}\right] - (s-0.1s)\theta\frac{at_0}{1-\theta t_0}$$
$$+ sI_e\left[\left(at_0 + \frac{bt_0{}^2}{2}\right) + (M - t_0)(a + bt_0)\right]\Bigg] + \frac{bc_1}{3} < 0 \qquad ------(27)$$

$$\frac{\partial^2 TP_2}{\partial t_0{}^2} = \frac{1}{T}\left[-P[2a\theta - b] - h(a + 3a\theta t_0 - 6a\theta t_0{}^2) - c_1[bT - 2bt_0] - (s-0.1s)2\theta^2 a\right.$$
$$\left. - PI_p[a + 6a\theta t_0 - 2a\theta M - \theta] - sI_e b\right] < 0 \qquad ------(28)$$

Provided that equation (19) and (20) satisfies the following conditions:

$$\left[\frac{\partial^2 \mathrm{TP_i}}{\partial \mathrm{t_0}^2}\right]\left[\frac{\partial^2 \mathrm{TP_i}}{\partial \mathrm{T}^2}\right]-\left[\frac{\partial^2 \mathrm{TP_i}}{\partial \mathrm{t_0}\,\partial \mathrm{T}}\right]<0 \qquad i=1,2.$$

4. NUMERICAL EXAMPLES

4.1 Example-I

Let A=85, a=200, b=0.05, P=20, s=30, h=0.16, c_1=4, θ=0.02, I_e=0.12, I_P=0.15, M=0.14 in appropriate units. The optimal value of t_0=0.4(0.4138), T= 0.8, Q=83(82.6779) and TP_1=2133.8 per unit.

4.2 Example-II

Let A=85, a=200, b=0.05, P=20, s=30, h=0.16, c_1=4, θ=0.02, I_e=0.12, I_P=0.15, M=0.35 in appropriate units. The optimal value of t_0= 0.4(0.4034), T=0.8, Q=81(80.6451) and TP_1=2128.1 per unit.

5. CONCLUSION

In this paper, we have developed a deterministic inventory model for deteriorating items in the presence of trade credit period under permissible delay in payments. In this model, the optimal cycle time and optimal order quantity and total relevant profit are calculated. Finally, numerical examples are illustrated.

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